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A method of determining surface friction in a turbulent boundary layer using plane Pitot tubes is described.

Plane Pitot tubes are widely used to measure the velocity distribution in a boundary layer. However, in order to represent the measured velocity profile in universal coordinates

$$
\begin{equation*}
u / u_{*}=f\left(y u_{*} / v\right), \tag{1}
\end{equation*}
$$

widely used in semiempirical theories of the turbulent boundary layer, it is necessary to develop a method for simultaneously measuring the velocity distribution and the value of the tangential stress on the wall $\tau_{w}=\rho u_{*}^{2}$ using one and the same plane Pitot tube. In addition, the use of a plane Pitot tube to measure friction instead of a circular Preston tube [1], which is widely employed in practice, considerably extends the experimental possibilities of determining friction using a Pitot tube. The fact is that measurements of friction using a circular Preston tube is only possible under nongradient-flow conditions due to its comparatively large dimensions. When there is a longitudinal pressure gradient in the flow, the boundary-layer region to which relation (1) (on which Preston's method is based) applies is considerably reduced and becomes comparable with the thickness of the viscous sublayer. It is shown in [2] that for large values of the longitudinal pressure gradient, measurement of the friction using a Pitot tube placed on the wall is only possible if the tube does not lie outside the viscous sublayer where a linear velocity distribution is observed, which is true, as experiments show, for all values of the longitudinal pressure gradient. The dimensions of plane tubes can be chosen in practice so that they satisfy the above requirement.

Table 1 shows the dimensions of the plane tubes investigated. Tubes with a ratio $B / H<$ 3 were not considered since they do not have any particular advantages over round tubes. From technological considerations, plane tubes have a strictly rectangular receiving opening. To estimate the effect on the readings of the plane tube of a possible deviation of the shape of the receiving opening from rectangular one of the tubes had rounded corners (tube No. 6).

The monitored value of the tangential stress on the wall ( $\tau_{w}$ ) was found by direct weight measurement of the friction using a "smoothing element." To do this we used magnetoelectric [3] and inductive [4] weights, which gave a reliable measurement of small frictional forces from 5. $10^{-6}$ to $10^{-3} \mathrm{~N}$. The readings of the plane tube were read on a precision alcohol manometer, on which the mean-square measurement error was 0.004 mm of water. The level of the alcohol was followed, read out, and recorded automatically [5].

Figure 1 compares the results of the calibration of the plane tubes with different geometrical dimensions with a calibrated Patel curve [6] for circular tubes (the continuous line) in Preston coordinates [1] for $D=H$

$$
\frac{H^{2} \tau_{w}}{4 \rho_{w o} v^{2}}=\varphi\left(\frac{H^{2} \Delta P}{4 \rho_{w} v^{2}}\right) .
$$

As can be seen, for plane tubes there is a large spread among the experimental points, and no definite relationship between the distribution of the experimental points and the geometrical dimensions of the tubes is observed.

Obtaining a single calibration relationship for plane tubes with different geometrical dimensions involves choosing a dimensionless geometrical parameter, which would take into account the two characteristic dimensions of the receiving opening of the plane tube (instead of one for a circular tube) and which would enable the spread between the experimental points to be reduced considerably.

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TABLE 1. Dimensions of the Plane Tubes

| Tube <br> No. | Notation <br> used in <br> Figs. 1,3 | $H, \mathrm{~mm}$ | $B, \mathrm{~mm}$ | $\frac{H}{B}$ | $\frac{D_{h}}{d_{h}}$ | $\delta_{1}, \mathrm{~mm}$ | $o_{2}, \mathrm{~mm}$ | $o_{2}, \mathrm{~mm}$ | $\delta_{4}, \mathrm{~mm}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0,2095 | 1,350 | 0,155 | 1,69 | 0,053 | 0,041 | 0,109 | 0,061 |
| 2 | 2 | 0,213 | 1,958 | 0,109 | 1,86 | 0,040 | 0,039 | 0,065 | 0,070 |
| 3 | 3 | 0,289 | 1,321 | 0,219 | 1,41 | 0,051 | 0,047 | 0,105 | 0,091 |
| 4 | 4 | 0,293 | 1,930 | 0,152 | 1,60 | 0,059 | 0,064 | 0,043 | 0,059 |
| 5 | 5 | 0,402 | 1,340 | 0,305 | 1,49 | 0,074 | 0,074 | 0,069 | 0,093 |
| 6 | 6 | 0,187 | 1,553 | 0,120 | 1,67 | 0,038 | 0,037 | 0,160 | 0,059 |



Fig. 1


Fig. 2

Fig. 1. Comparison of the results of a calibration of the plane tubes with a Patel calibration curve [6] for circular tubes. The notation is shown in Table 1.

Fig. 2. Dependence of the coefficient $k$ on the ratio $B / H$.
Moreover, from the practical point of view it is extremely important to obtain a universal calibration relation suitable both for plane and for circular tubes, which would facilitate the wider introduction of plane tubes for measuring friction.

We tested several different expressions for the geometrical function $\overline{\mathrm{H}}(\mathrm{B}, \mathrm{H})$, including power functions of the ratio $B / H$ and of the ratio of the hydraulic diameters of the plane tube with respect to the external and internal contour of the receiving opening $\mathrm{D}_{\mathrm{h}} / \mathrm{d}_{\mathrm{h}}$, but the most successful relationship was the one for $\overline{\mathrm{H}}(\mathrm{B}, \mathrm{H})$ which is obtained if one equates the effective area of the receiving opening of the plane tube to the area of the receiving opening of the circular tube of diameter $D=H$ [7], i.e.,

$$
k B H=\frac{\pi \bar{H}^{2}}{4}
$$

whence

$$
\begin{equation*}
\bar{H}=H \sqrt{\frac{4 k}{\pi}\left(\frac{B}{H}\right)} \tag{2}
\end{equation*}
$$

Here the coefficient $k$ takes into account the particular features of the flow around the plane tube compared with the flow around a circular tube and is determined by the requirement that the calibration curves for the circular and plane tubes should be identical. Then we have

$$
k=\frac{\pi}{4} \frac{H}{B} \frac{\tau_{w} D^{2} / 4 \rho v^{2}}{\tau_{w} H^{2} / 4 \rho v^{2}}
$$



Fig. 3. Comparison of the results of an analysis of the calibration of plane tubes using transformation (2) with the Patel calibration curves [6] for circular tubes. The notation is shown in Table 1.

Analysis of the experimental data shown in Fig. I showed that over a large part of the boundary layer the value of $k$ is independent of the flow conditions in the boundary layer, and is determined solely by the relative dimensions of the receiving area of the same tube $H / B$ (Fig. 2). The small circles in Fig. 2 indicate the limiting value of $k=\pi / 4$, which would be obtained if we assume that the results of the measurements on tubes with a square receiving area ( $B / H=1$ ) correspond to the results of measurements with a circular tube for $H=\bar{H}$.

However, in the immediate vicinity of the walls, viz., when $\tau_{W} \bar{H}^{2} / 4 \rho v^{2}<40$, which corresponds to $y u_{*} / v<6.3(y=H / 2)$, i.e., in that part of the viscous sublayer where there is a linear vellcity distribution, the value of $k$ is not a unique function of $H / B$ and depends to some extent on the flow conditions in the boundary layer.

Figures 3 shows the results of a calibration of plane tubes in $\Delta \mathrm{PH}^{2} / 4 \rho \nu^{2}=f\left(\tau_{w} \overline{\mathrm{H}}^{2} / 4 \rho \nu^{2}\right)$ coordinates using the values of $k$ given in Fig. 2. It can be seen that the spread among the experimental points due to different dimensions of the receiving area of the plane tubes is considerably reduced over the whole thickness of the boundary layer including the viscous sublayer: Over a large part of the boundary layer all the experimental points lie on the Patel curve [6] for a circular tube. The experimental points relating to the region near the walls of the viscous sublayer with a linear velocity distribution lie somewhat higher than the Patel curve. It is worth noting that the rounding of the corners in the plane tube (tube No. 6) did not lead to any appreciable departure of its reading from the readings of the rectangular tube.

For $40<\tau_{\mathrm{w}} \overline{\mathrm{H}}^{2} / 4 \rho^{2}<164$, the experimental points for plane tubes with different dimensions of the receiving area can be described by the Patel calibration relation [6] for circular tubes taking the above transformation into account, viz.,

$$
\begin{equation*}
\lg \frac{\tau_{v} \bar{H}^{2}}{4 \rho v^{2}}=0.8287-0.1381 \lg \frac{\Delta P H^{2}}{4 \rho v^{2}}+0.1437\left(\lg \frac{\Delta P H^{2}}{4 \rho v^{2}}\right)^{2}-0.006\left(\lg \frac{\Delta P H^{2}}{4 \rho v^{2}}\right)^{3} \tag{3}
\end{equation*}
$$

In the region of the viscous sublayer when $\tau_{W} \bar{H}^{2} / 4 \rho \nu^{2}<40$, the experimental points are described by the empirical relation

$$
\begin{equation*}
\lg \frac{\tau_{w} \overline{H^{2}}}{4 \rho v^{2}}=0.68 \lg \frac{\Delta P H^{2}}{4 \rho v^{2}}-0.493 \tag{4}
\end{equation*}
$$

The values of $\overline{\mathrm{H}}$ in (3) and (4) are found from (2) and the relation


Fig. 4. Comparison of the results of the experimental determination of $\tau_{w}\left(N / m^{2}\right)$ for $d P / d x \neq 0$ using a plane tube and a control method: 1) plane tube, $H=0.187 ; 2$ ) thermal friction sensor [2].

$$
k=0.68(H / B)
$$

The experimental fact that in the immediate vicinity of the walls in the viscous-sublayer region the linear velocity distribution remains true for all values of the longitudinal pressure gradient [2], enables us to use relation (4) for thin plane tubes both for gradient-free flow of a liquid and gas and when $\mathrm{dP} / \mathrm{dx} \neq 0$ (Fig. 4).

## NOTATION

$u$, flow velocity at a distance $y$ from the wall; $u *=\sqrt{\tau_{w} / p}$, friction velocity; $\Delta \mathrm{P}$, difference between the total pressure measured by the tube and static pressure on the wall; $\rho$, air density; $v$, kinematic air viscosity; $\Delta=\left(\nu / \rho u_{\hbar}^{3}\right) \mathrm{dP} / \mathrm{dx}$, pressuregradient parameter; $\mathrm{D}_{\mathrm{h}}=2$ $\mathrm{HB} /(\mathrm{H}+\mathrm{B})$; and $\mathrm{d}_{\mathrm{h}}=2 \mathrm{hb} /(\mathrm{h}+\mathrm{b})$.

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